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Conformal symmetry of superstrings on $AdS_3 \times S^3 \times T^4$ and D1/D5 system

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Abstract

Conformal field theory of the D1/D5 system and superstrings on $AdS_3 \times S^3 \times T^4$ is studied with particular attention to the world-sheet fields corresponding to the T^4 part. A solution to the spacetime $N = 4$ superconformal symmetry doubling and other problems is proposed. It is argued that the relevant spacetime symmetry should be based on the middle $N = 4$ superconformal algebra. It is discussed as to why this superconformal structure has been missed so far.

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1 Introduction

It was known for some time that there exist two commuting copies of infinite dimensional conformal symmetry in any theory of three dimensional gravity with negative cosmological constant at the boundary of the spacetime[1]. Interest in this subject was renewed because of the AdS/CFT correspondence enunciated in [2] and made more precise later [3, 4]. This has lead to some remarkable advances, for example, in the study black holes[5]-[7] and of Wilson loops (see [8] for a review). Examination of this correspondence from the world sheet, Green-Schwarz formalism and D1/D5 system points of view has lead to even better understanding [9]-[15].

A recurring theme in these developments is the presence of higher superconformal symmetries. Particularly both small and large $N = 4$ symmetries are known to occur. This happens, for example, in case of superstrings moving on $\mathcal{M} = AdS_3 \times S^3 \times T^4$ and $\mathcal{N} = AdS_3 \times S^3 \times S^3 \times S^1$ manifolds, respectively.

Though this study has clarified many issues but some important details have been missed. This manifests itself in the following curious problem. We know that a condition to get spacetime supersymmetry is the occurrence of $N = 2$ world-sheet supersymmetry. This requires the existence of a conserved $U(1)_R$ current on the world-sheet under which the supercurrents $G^{\pm}$ have charges ± 1. It turns out that one can not tread this standard route in case of the superstrings on $\mathcal{M}$. This was pointed out in reference [9]. It was observed there that this route leads to some serious problems of interpretation of spacetime supersymmetry. It was found that this analysis gives two copies of Ramond sector supercharges. This leads, in turn, to two small $N = 4$ superconformal symmetries on the boundary of $\mathcal{M}$. This is puzzling and unexpected. Normally only one such symmetry should exist.

One would like to understand this occurrence of two copies of the spacetime boundary symmetry. These two copies can not be regarded as left and right sectors of the spacetime theory due to several reasons. The world-sheet and spacetime chiralities are related and therefore a single chirality on the world-sheet can not lead to both chiralities on the spacetime boundary. Moreover the other chirality on the world-sheet will still give more spacetime symmetry. Other interpretations of the second copy of the small $N = 4$ superconformal symmetry are equally inconvenient.

There is another puzzling feature of the standard world-sheet free field analysis of the AdS/CFT correspondence in case of the manifold $\mathcal{M}$. Two

$U(1)$ currents are available on the spacetime boundary which can be used to generate the $SU(2)$ Kac-Moody symmetry required to generate the small $N = 4$ superconformal symmetry. There is an inbuilt asymmetry in the choice of $U(1)$ currents because only one of them is chosen by internal structure. There should be a natural explanation of this observation.

These problems are addressed in this note. It turns out that all of these problems are related to each other. A closer look at the structure of the background manifold and the field content of the theory leads to their resolution. It is convenient to begin with the free field analysis of the CFT of long string in the context of D1/D5 system or rather its S-dual NS1/NS5. Eventually one gets a single copy of the *midlle* rather than two copies of the small $N = 4$ superconformal algebra. We argue that the standard analysis using world-sheet free field approach to strings propagating on the manifold $\mathcal{M}$ should lead to the same conclusion. We also discuss as to how this aspect has remained undetected.

Rest of this papers is organised as follows. In Section 2 we study the CFT of the long string on the manifold $\mathcal{M}$ using the free field analysis in the context of D1/D5 system. Comments on the usual world-sheet free field analysis are contained in Section 3. We end this paper with a discussion in Section 4.

2 CFT of the long string

In reference [14], apart from other things, the D1/D5 system, constructed from branes in $R^6 \times T^4$, was studied. If the numbers of D1 and D5 branes are Q_1 and Q_5 respectively then for large value of Q_1 the D1/D5 system is described by a conformal field theory on the boundary of $AdS_3 \times S^3$. This is the conformal field theory of the large string. The free field realization of the corresponding S-dual theory, the NS1/NS5 system, was obtained in above reference. We start with a review of these results and then take a closer look at the symmetry generated by the free fields corresponding to the T^4 part of $\mathcal{M}$.

Expression for the generators of this conformal theory are given below.

$$\begin{aligned} T &= -\frac{1}{2}\partial S^\mu S^\mu - \frac{j^a j^a}{Q_5} - \frac{1}{2}\partial\phi\partial\phi + \frac{Q_5-1}{\sqrt{2Q_5}}\partial^2\phi \\ G^\mu &= \frac{1}{\sqrt{2}}\partial\phi S^\mu - \frac{2}{\sqrt{Q_5}}\eta^a_{\mu\nu} j^a S^\nu + \frac{1}{6\sqrt{Q_5}}\epsilon_{\mu\nu\rho\sigma} S^\nu S^\rho S^\sigma - \frac{Q_5-1}{\sqrt{Q_5}}\partial S^\mu \end{aligned}$$

$$J^a = j^a + \frac{1}{2}\eta^a_{\mu\nu}S^\mu S^\nu. \tag{1}$$

Here the indices $\mu, \nu, \cdots$ take four values from 0 to 3 and $a, b, \cdots$ take three values from 1 to 3. These operators satisfy the following OPEs of the small $N = 4$ superconformal algebra with $c = 6(Q_5 - 1)$

$$\begin{aligned}
G^\mu(z)G^\nu(w) &\sim \frac{2\delta^{\mu\nu}(Q_5-1)}{(z-w)^3} - \frac{4\eta^a_{\mu\nu}J^a}{(z-w)^2} + \frac{\left(\delta^{\mu\nu}T - 2\eta^a_{\mu\nu}\partial J^a\right)}{z-w} \\
J^a(z)J^b(w) &\sim -\frac{\delta^{ab}(Q_5-1)/2}{(z-w)^2} + \frac{\epsilon^{abc}J^c}{z-w} \\
T(z)J^a(w) &\sim \frac{J^a}{(z-w)^2} + \frac{\partial J^a}{z-w}\,, \qquad J^a(z)G^\mu(w) \sim \frac{\eta^a_{\mu\nu}}{z-w}G^\nu \\
T(z)T(w) &\sim \frac{3(Q_5-1)}{(z-w)^4} + \frac{2T}{(z-w)^2} + \frac{\partial T}{z-w} \\
T(z)G^\mu(w) &\sim \frac{\frac{3}{2}G^\mu}{(z-w)^2} + \frac{\partial G^\mu}{(z-w)}
\end{aligned} \tag{2}$$

where 't Hooft η and $\bar{\eta}$ symbols are defined as

$$\begin{aligned}
\eta^a_{\mu\nu} &= \alpha^{+a}_{\mu\nu} = \frac{1}{2}(\delta_{a\mu}\delta_{0\nu} - \delta_{a\nu}\delta_{0\mu} + \epsilon_{a\mu\nu}), \\
\bar{\eta}^a_{\mu\nu} &= \alpha^{-a}_{\mu\nu} = \frac{1}{2}(\delta_{a\nu}\delta_{0\mu} - \delta_{a\mu}\delta_{0\nu} + \epsilon_{a\mu\nu}).
\end{aligned} \tag{3}$$

OPEs (2) can be verified using the two point functions

$$\begin{aligned}
S^\mu(z)S^\nu(w) &\sim -\frac{\delta^{\mu\nu}}{z-w}, \qquad \partial\phi(z)\partial\phi(w) \sim -\frac{1}{(z-w)^2} \\
j^a(z)j^b(w) &\sim -\frac{\delta^{ab}(Q_5-2)/2}{(z-w)^2} + \frac{\epsilon^{abc}j^c}{z-w}.
\end{aligned} \tag{4}$$

Apart from this symmetry one may construct one more set of generators realizing another small $N = 4$ superconformal symmetry using the free fields corresponding to the T^4 part of the manifold. This realization will have a central charge $c = 6$. This takes the total of the central charge to $6Q_5$.

A convenient path to proceed further is to write down the generators of the small $N = 4$ corresponding to T^4 part of the manifold. Since we require $c = 6$ therefore putting $Q_5 = 2$ in (1) will give us one possible realization.

Let us denote the corresponding "free" fields by the symbols φ, $\tilde{j}^a$ and $\tilde{S}^\mu$. Using these fields and Q_5 put equal to 2 one gets the following expressions for the generators.

$$\begin{aligned}
\tilde{T} &= -\frac{1}{2}\partial\tilde{S}^\mu\tilde{S}^\mu - \frac{1}{2}\tilde{j}^a\tilde{j}^a - \frac{1}{2}\partial\varphi\partial\varphi + \frac{1}{2}\partial^2\varphi \\
\tilde{G}^\mu &= \frac{1}{\sqrt{2}}\partial\varphi\tilde{S}^\mu - \sqrt{2}\eta^a_{\mu\nu}\tilde{j}^a\tilde{S}^\nu - \frac{1}{6\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}\tilde{S}^\nu\tilde{S}^\rho\tilde{S}^\sigma - \frac{1}{\sqrt{2}}\partial\tilde{S}^\mu \\
\tilde{J}^a &= \tilde{j}^a + \frac{1}{2}\eta^a_{\mu\nu}\tilde{S}^\mu\tilde{S}^\nu
\end{aligned} \tag{5}$$

The two point functions for the new fields are

$$\begin{aligned}
\tilde{S}^\mu(z)\tilde{S}^\nu(w) &\sim -\frac{\delta^{\mu\nu}}{z-w}, \qquad \partial\varphi(z)\partial\varphi(w) \sim -\frac{1}{(z-w)^2} \\
\tilde{j}^a(z)\tilde{j}^b(w) &\sim \frac{\epsilon^{abc}\tilde{j}^c}{z-w}.
\end{aligned} \tag{6}$$

Though the operators (5) obey the small $N = 4$ superconformal algebra with central charge $\tilde{c} = 6$ but there are some serious shortcomings in this symmetry. First of all the algebra so generated has an $SU(2)$ Kac-Moody symmetry. This is expected for the internal structure of the $N = 4$ symmetry but not acceptable because T^4 does not have an $SU(2)$ isometry. Even if we devise a way of avoiding this problem then the $U(1)^4$ affine symmetry expected for the present manifold T^4 is missing. Boson φ is anomalous and we do not have even a single independent $U(1)$ affine symmetry apart from the one occurring as the Cartan subalgebra of $SU(2)$. Another awkward feature is that the level of the $SU(2)$ currents $\tilde{j}^a$ is zero.

First one of these problems, that is, the existence of an extra $SU(2)$ is an indication that the algebra generated by the generators (5) does not have an independent existence. The fields occurring in eqn.(5) must be assimilated in the over all symmetry algebra. We shall do so in a shortwhile.

Two things have to be done to to solve the second problem. First of all we must find a realization in which the boson φ is not anomalous so that it can give us a $U(1)$ current. Secondly we should drop the classical currents $\tilde{j}^a$ which are not serving very useful purpose here. In there place we must introduce three more free bosons φ^a to get three other $U(1)$ currents. Thus we will have four free bosons φ^μ, such that $\varphi = \varphi^0$, and four free fermios $\tilde{S}^\mu$ which we must use to get a free field realization of the small algebra with

central charge 6. Such a realization is easy to write is is already known. Expressions of the generators in this realization are given below.

$$\begin{aligned}
\check{T} &= -\frac{1}{2}\partial\tilde{S}^\mu\tilde{S}^\mu - \frac{1}{2}\partial\varphi^\mu\partial\varphi^\mu \\
\check{G}^\mu &= \frac{1}{\sqrt{2}}\partial\varphi\tilde{S}^\mu - \sqrt{2}\eta^a_{\mu\nu}\partial\varphi^a\tilde{S}^\nu \\
\check{J}^a &= \frac{1}{2}\eta^a_{\mu\nu}\tilde{S}^\mu\tilde{S}^\nu.
\end{aligned} \tag{7}$$

These generators realize a small $N = 4$ superconformal symmetry with central charge 6. In addition to that there are four $U(1)$ currents given by $U^\mu \sim \partial\varphi^\mu$, all non-anomalous. That is not all. A comparison with eqn.(16) of reference [16] reveals that eqs.(7) give a realization of the middle $N = 4$ superconformal algebra, a symmetry more stringent than small one[17]-[18]. This, of course, does not mean that we have a middle superconformal symmetry on T^4. This is because this realization too has an $SU(2)$ Kac-Moody symmetry while there is no $SU(2)$ isometry on T^4. (Of course this argument will apply only if the naive expectation that the all the Kac-Moody symmetries on the AdS boundary should come from an isometry. In this connection one should remember that AdS_3 part leads to the infinite Virasoro symmetry on the boundary of the spacetime.)

The complete spacetime superconformal symmetry on $\mathcal{M}$ will be a suitable combination of the generators (1) and (7). In this combination only the $SU(2)$ affine symmetry of (1) and $U(1)^4$ of (7) should be present not the $SU(2)$ of (7). In other words our problem is reduced to the following one. We have a set of field ϕ, φ^μ, S^μ, $\tilde{S}^\mu$ and $SU(2)$ currents j^a. Using them we have to write a free field realization which must have at least small $N = 4$ superconformal symmetry, an $SU(2)$ current algebra and four $U(1)$ symmetries. Answer to this problem is already known. These conditions lead to a realization of the middle $N = 4$ superconformal algebra. This is given in eqn.(23) of ref.[16]. After some rescalings and suitable redefinitions this realization can be written as follows.

$$\begin{aligned}
\hat{T}(z) &= -\frac{1}{2}(\partial\phi)^2 - \frac{Q_5 - 1}{\sqrt{2Q_5}}\partial^2\phi - \frac{1}{Q_5}j^a j^a - \frac{1}{2}\partial\varphi^\mu\partial\varphi^\mu \\
&\quad -\frac{1}{2}S^\mu\partial S^\mu - \frac{1}{2}\tilde{S}^\mu\partial\tilde{S}^\mu, \\
\hat{U}^\mu(z) &= \partial\varphi^\mu(z), \qquad \hat{Q}^\mu(z) = \frac{1}{\sqrt{2}}\tilde{S}^\mu(z),
\end{aligned}$$

$$
\begin{aligned}
\hat{J}^1(z) &= j^1 - \frac{1}{2}S^0S^1 + \frac{1}{2}S^2S^3 - \frac{1}{2}\tilde{S}^0\tilde{S}^1 + \frac{1}{2}\tilde{S}^2\tilde{S}^3, \text{cyclic for } \hat{J}^{2,3}, \\
\hat{G}^0(z) &= \frac{1}{\sqrt{2}}\partial\phi S^0 - \frac{Q_5-1}{\sqrt{Q_5}}\partial S^0 + \frac{1}{\sqrt{2}}\partial\varphi^\mu\tilde{S}^\mu \\
&\quad + \frac{1}{\sqrt{Q_5}}(j^aS^a + S^1S^2S^3), \\
\hat{G}^1(z) &= \frac{1}{\sqrt{2}}\partial\phi S^1 - \frac{Q_5-1}{\sqrt{Q_5}}\partial S^1 + \frac{1}{\sqrt{2}}\partial\varphi\tilde{S}^1 \\
&\quad + \frac{1}{\sqrt{Q_5}}(-j^1S^0 + j^2S^3 - j^3S^2 - S^0S^2S^3) \\
&\quad + \frac{1}{\sqrt{2}}(-\partial\varphi^1\tilde{S}^0 + \partial\varphi^2\tilde{S}^3 - \partial\varphi^3\tilde{S}^2).
\end{aligned} \tag{8}
$$

with cyclic expressions for $\hat{G}^2(z)$ and $\hat{G}^3(z)$. These operators satisfy the OPEs of the middle $N=4$ superconformal algebra. These OPEs include all the ones in eqn.(2) with T, G, J^a and Q_5-1 replaced by $\hat{T}$, $\hat{G}$, $\hat{J}^a$ and Q_5 respectively. Additional OPEs are given below.

$$
\begin{aligned}
\hat{T}(z)\hat{U}^\mu(w) &= \frac{\hat{U}^\mu}{(z-w)^2} + \frac{\partial\hat{U}^\mu}{z-w}, \\
\hat{T}(z)\hat{Q}^\mu(w) &= \frac{\hat{Q}^\mu/2}{(z-w)^2} + \frac{\partial\hat{Q}^\mu}{z-w}, \\
\hat{U}^a(z)\hat{G}^\mu(w) &= \frac{\eta^a_{\mu\nu}\hat{Q}^\nu}{(z-w)^2}, \qquad \hat{U}^\mu(z)\hat{U}^\nu(w) = -\frac{\delta^{\mu\nu}}{(z-w)^2}, \\
\hat{J}^a(z)\hat{U}^\mu(w) &= 0 = \hat{U}^\mu(z)\hat{Q}^\nu, \qquad \hat{U}(z)\hat{G}^\mu(w) = \frac{\hat{Q}^\mu}{(z-w)^2}, \\
\hat{Q}^\mu(z)\hat{G}^\nu(w) &= \frac{\eta^a_{\mu\nu}\hat{U}^a - \delta^{\mu\nu}\hat{U}/2}{z-w}, \\
\hat{J}^a(z)\hat{Q}^\mu(w) &= \frac{\eta^a_{\mu\nu}\hat{Q}^\nu}{z-w}, \quad \hat{Q}^\mu(z)\hat{Q}^\nu(w) = -\frac{\delta^{\mu\nu}/2}{z-w}.
\end{aligned} \tag{9}
$$

Here $\hat{U}(z) = \hat{U}^0(z)$ and the central charge of the realization is $\hat{c} = 6Q_5$.

Thus with this analysis we conclude that for the manifold $\mathcal{M}$ the D1/D5 system should be described by a middle rather than the small $N=4$ superconformal symmetry. Particularly we need not think of the symmetry to be consisting of two small $N=4$ parts corresponding to $AdS_3 \times S^3$ and T^4 factors.

3 Comments on the world-sheet analysis of superstrings on $\mathcal{M}$

It is well known that superstring propagating on $\mathcal{N}$ lead to the large $N = 4$ superconformal symmetry. In the usual world-sheet analysis this was shown in [10]. Another well known fact is that Inönü-Wigner contraction of the large $N = 4$ superconformal symmetry leads to the middle one[17, 18]. In the corresponding limit on the group manifold we go from $\mathcal{N}$ to $\mathcal{M}$. If we do a straightforward extrapolation of the results of the analysis in the last section we again reach the same conclusion. But the direct construction of the symmetry on the boundary of the spacetime in case of the superstrings on $\mathcal{M}$ have so far lead to the small $N = 4$ superconformal structure only. How did we miss the middle $N =$ superconformal structure? Then there is the serious problem of interpreting the results of the standard world-sheet approach to spacetime supersymmetry described in the Appendix B of reference [9].

These two problems are complementary to each other. If we assume that the actual spacetime superconformal symmetry of the superstrings on $\mathcal{M}$ is the middle one then the second problem has a natural solution. In this section we take the first one and deal with the second one in the next section.

To answer the first problem raised above we start with the following observation. The usual world-sheet analysis of strings on AdS_3 manifolds gives us most easily the global part of the spacetime superconformal symmetry. Some of the (anti-) commutators of this algebra are

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n}, \quad [L_m, \Phi_n] = [(d_\Phi - 1)m - n]\Phi_{m+n}, \\
&\quad \Phi_n \in \{G^\mu_n, A^{\pm a}_n, U_n, Q^\mu_n\}, d_\Phi \in \{3/2, 1, 1, 1/2\}, \\
\{G^\mu_r, G^\nu_s\} &= \delta^{\mu\nu} L_{r+s} + 2(r-s)[\gamma\alpha^{+a}_{\mu\nu} A^{+a}_{r+s} + (1-\gamma)\alpha^{-a}_{\mu\nu} A^{-a}_{r+s}], \\
\left[A^{\pm a}_0, G^\mu_r\right] &= \alpha^{\pm a}_{\mu\nu} G^\nu_r, \quad \left[A^{\pm a}_0, A^{\pm b}_0\right] = \epsilon^{abc} A^{\pm c}_0, \left[A^{-a}_0, A^{+b}_0\right] = 0, \\
[U_0, G^\mu_r] &= 0, \quad \left[U_0, A^{\pm a}_0\right] = 0, \quad [U_0, U_0] = 0
\end{aligned} \tag{10}$$

and the rest of them are

$$\begin{aligned}
\{Q^\mu_r, G^\nu_s\} &= \alpha^{+a}_{\mu\nu} A^{+a}_{r+s} - \alpha^{-a}_{\mu\nu} A^{-a}_{r+s} + (\delta^{\mu\nu}/2) U_{r+s}, \\
\left[A^{\pm a}_0, Q^\mu_r\right] &= \alpha^{\pm a}_{\mu\nu} Q^\nu_r, \quad [U_0, Q^\mu_r] = 0, \\
\{Q^\mu_r, Q^\nu_s\} &= -\frac{c}{12\gamma(1-\gamma)} \delta^{\mu\nu} \delta_{r+s,0},
\end{aligned} \tag{11}$$

where $A^{\pm a}$ are the two $SU(2)$ currents and γ is a parameter related to the levels of the to $SU(2)$ algebras.

The standard analysis on the world-sheet takes us to the (anti-) commutators (10)[10]. At the same time this part misses the (anti-) commutators crucial for the middle $N = 4$ structure when one goes from the manifold $\mathcal{N}$ to $\mathcal{M}$. Particularly dimension half fermionic generators are completely missing from eqn.(10). Thus contraction of (10) will easily lead to the small $N = 4$ algebra but will not know about the remaining structure relevant for the middle algebra. The small algebra is a subalgebra of the middle one and looking at it will not lead to the larger algebra. Thus to get the complete and real symmetry of the theory one has to explore beyond the small algebra. One way to remedy the situation is to start with the full $N = 4$ superconformal algebra rather than the partial set (10) of the (anti-) commutators. Then the middle $N = 4$ superconformal algebra is obtained by an Inönü-Wigner contraction [17, 18]. At present this strategy can be implemented for the Wakimoto type of realization[19].

4 Discussion

A natural conclusion of the observations made in the earlier Sections is that the spacetime symmetry for the superstrings moving on the manifold $\mathcal{M}$ and the conformal field theory of the related D1/D5 should be based on the middle $N = 4$ superconformal algebra. This is relevant in its own right because the T^4 part of the manifold should play its role in deciding the symmtery structure. In addition to that it should lead to a solution of the problems encountered in the standard world-sheet analysis of superstrings on $\mathcal{M}$.

Doubling of the spacetime superconformal symmetry disappears if the generators of one of the copies of the symmetry are rehashed to enlarge the other copy to the level of middle algebra. Number counting is certainly in favour of this proposal. The small algebra has eight generators while the middle one has sixteen of them. Out of the eight spin fields in the Green-Schwarz analysis four should lead to the four supercharges and four to the other four fermionic dimension half generators. (Same thing should happen even in the case of the superstrings moving on $\mathcal{N}$ regarding the expressions for the dimension half generators.) The problem of asymmetry in selection of the $U(1)$ current, mentioned in the Introduction, also has a natural explanation.

The $U(1)$ not required for the extension to $SU(2)$ level is one of the four $U(1)$ currents required for the middle algebra. Question of chirality mixing does not arise at all.

An open problem in this regard is the explicit construction of the generators relevant for the middle structure in the same way as it has been done for the other generators in the old fashioned world-sheet analysis. Problem for the D1/D5 system was easy because things really came down to the level of simple free field theories if we use the approach pioneered in reference [14]. For the superstrings on $\mathcal{M}$ or $\mathcal{N}$ the problem is compounded by the difficulty in the construction of the dimension half generators using the spin fields. It will be more illuminating if one can find a technique to promote the spin fields to the level of free fields on the AdS_3 boundary parallel to the approach in case of D1/D5 system. It certainly will be more transparent than the standard procedure of bosonosing the spin fields and then constructing fermionic operators out of the bosons thus obtained.

Next step in this work will be to analyze the spectrum of the superstrings moving on $\mathcal{M}$ in the light of the symmetry described above. Even before that one should study the representations of the middle $N = 4$ superconformal symmetry itself. We hope to return to these issues in future.

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